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ELECTRIC CHARGE ESTIMATION OF A NEW-BORN BLACK HOLE

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Though a black hole can theoretically possess a very big charge ($Q/(\sqrt{GM}) \simeq 1$), the charge of the real astrophysical black holes is usually considered to be negligible. This supposition is based on the fact that an astrophysical black hole is always surrounded by some plasma, which is a very good conductor. However, it disregards that the black holes have usually some angular momentum, which can be interpreted as its rotation of a sort. If in the plasma surrounding the hole there is some magnetic field, it leads to the electric field creation and, consequently, to the charge separation.

In this article we estimate the upper limit of the electric charge of stellar mass astrophysical black holes. We have considered a new black hole formation process and shown that the charge of a new-born black hole can be significant ($\sim 10^{13}$ Coulombs). Though the obtained charge of an astrophysical black hole is big, the charge to mass ratio is small $Q/(\sqrt{GM}) \sim 10^{-7}$, and it is not enough to affect significantly either the gravitational field of the star or the dynamics of its collapse.

Keywords: Black Hole; Electric Charge; Classical Electrodynamics.

1. Introduction

The prediction and the discovery of stellar mass black holes certainly were a great achievement of the last century. However, it is still unclear if astrophysical black holes can possess a significant electric charge. The main objection is related to common observation that there is no charge in the standard astrophysical objects like stars and planets. We know anyway that all these objects have an electric charge which is nonzero. As an example, the sun charge computed following Rosseiland (1924) is equivalent to a few hundred of coulombs. However, from the point

of view of relativistic astrophysics, it is insignificant. There is also a theoretical objection¹, related to the fact that a compact object with a net charge greater than $\sim 100 \times (M/M_\odot) C$, would be rapidly discharged by interaction with the surrounding plasma.

The critic remarks, we can address to the two objections, are the following:

1) Regarding the observations, we know that it is related to criteria and precision of the experiment. In fact, new space-based missions will be able to measure the position of stars with a precision of $1 \mu\text{as}$. This will be sufficient to scrutinize the possible gravitational lensing and evidence the part due to the charge of the black hole.

2) From the theoretical point of view, we know that the Reissner-Nordström metric allows a maximum charge given by : $Q_{max} = \sqrt{G} \times M \simeq 1.7 \cdot 10^{20} (M/M_\odot) C$. We want to stress that the objection¹ is not a theoretical limit since it depends on the modelization of the local environment and presupposes that this charge is static. We cannot exclude a peculiar situation when we could dynamically form a charged black hole that could evolve after formation to a complete or partial discharge. Here the key assumption is related to a dynamical approach. Without this fence of a static charge for an astrophysical object we can explore how a compact object could acquire a net charge. In this article, we propose an astrophysical scenario in order to obtain a charged black hole. Our basis is that we have observed numerous compact objects with a high magnetic field ($B \simeq 10^{13} \text{ Gs}$) and with high rotation velocity ($\Omega \simeq 10^3 \text{ rad.s}^{-1}$). The aim of this article is to show that there is no reason to believe that the charge of a new-born black hole is negligible.

2. Model description

As it is believed, stellar black holes are born most likely as a result of supernova explosions of massive stars². At the later stages of evolution this sort of a star consists of a compact and dense core and an extensive rare envelope. Finally, the core loses stability and collapses into a black hole. The envelope is erupted out and observed as a supernova phenomenon^a.

During all the process the collapsing core is surrounded by some plasma which is a very good conductor. Usually, it is considered as a critical argument that the core and, consequently, the new-born black hole cannot have any significant charge. However, it disregards that the core rotates and possesses a strong magnetic field. If the resistance of the substance is negligible, the electric field in the comoving frame of reference must be zero. In the static frame of reference the electric field is defined

^aStrictly speaking, there are other models of the stellar black hole creation, for instance, with no supernova explosion (so-called silent collapse²).

by the Lorentz transform:

$$\vec{E} = -\frac{\left[\frac{\vec{v}}{c} \times \vec{B} \right]}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (1)$$

$$\vec{E} = -\left[\frac{\vec{v}}{c} \times \vec{B} \right] \quad \text{in the nonrelativistic limit} \quad (2)$$

Generally speaking, the charge density $\rho \sim \text{div } \vec{E}$ is nonzero. Consequently, the core charge need not be obligatory equal to zero.

In order to estimate the charge of the core we use the following procedure:

- (1) Magnetic field calculation around the collapsing core.
- (2) Determination of the surface velocity and electric field calculation in accordance with (1).
- (3) The charge calculation from the Maxwell equation

$$Q = \frac{1}{4\pi} \oint \vec{E} \cdot d\vec{S} \quad (3)$$

We make several assumptions to simplify the solution. First of all, the core and the substance surrounding the core are supposed to have infinite conductivity (see substantiating calculations in the *Discussion*). We presume the gravitational field around the core to be the Schwarzschild one and the collapse to be spherically symmetric. Of course, for the rotating core it cannot be precisely correct. However, if the rotation is not very rapid, the deviation from the Schwarzschild metric should not be very big. Acceptability of the supposition will be discussed in the *Discussion*. We presume that the magnetic field has only the dipole component during the collapse and the axis of rotation coincides with the axis of the magnetic dipole. We make also some more assumptions, but they will be mentioned during the solution itself.

3. Calculations

Hereafter t , r , θ , and ϕ are the standard Schwarzschild coordinates, $r_g \equiv \frac{2GM}{c^2}$. We use the orthonormal tetrad frame

$$\left(\vec{e}_0 = \frac{1}{\sqrt{1 - r_g/r}} \partial_t, \quad \vec{e}_1 = \sqrt{1 - \frac{r_g}{r}} \partial_r, \quad \vec{e}_2 = \frac{1}{r} \partial_\theta, \quad \vec{e}_3 = \frac{1}{r \sin \theta} \partial_\phi \right) \quad (4)$$

The corresponding coframe of differential forms looks like:

$$\left(\omega^0 = \sqrt{1 - \frac{r_g}{r}} dt, \quad \omega^1 = \frac{dr}{\sqrt{1 - r_g/r}}, \quad \omega^2 = r d\theta, \quad \omega^3 = r \sin \theta d\phi \right)$$

With this choice of the frame the metric has the Lorentz form $g_{\alpha\beta} \equiv \eta_{\alpha\beta}$.

The quantities \vec{B} , \vec{E} , \vec{F} , and \vec{v} are three-vectors; their components are given in the spatial orthonormal triad set of basis three-vectors, corresponding to the chart (4).

$$\left(\vec{e}_r = \sqrt{1 - \frac{r_g}{r}} \partial_r, \quad \vec{e}_\theta = \frac{1}{r} \partial_\theta, \quad \vec{e}_\phi = \frac{1}{r \sin \theta} \partial_\phi \right)$$

3.1. Magnetic field calculation

The dipole magnetic field of a spherically symmetric massive body is described by the formulae^{3,4}:

$$B_r = A \left[-\ln \left(1 - \frac{r_g}{r} \right) - \frac{r_g}{r} - \frac{r_g^2}{2r^2} \right] \cos \theta \quad (5)$$

$$B_\theta = A \left[\left(\frac{r}{r_g} - 1 \right) \ln \left(1 - \frac{r_g}{r} \right) + 1 - \frac{r_g}{2r} \right] \sqrt{\frac{r_g^2}{r(r - r_g)}} \cdot \sin \theta \quad (6)$$

The equations are valid for all the space out of the core, but only the surface of the collapsing core is the object of our interest, and we mean by r hereafter the radius of the core. A is a coefficient proportional to the dipole momentum of the system.

In order to obtain the magnetic field around the collapsing core we assume⁴ the collapse to be quasistatic. Of course, this supposition is artificial, especially of the last stages of collapse, but it gives us the opportunity to proceed a strict examination of the problem in the framework of general relativity.

If the collapse is quasistatic, the magnetic field around the core is always described by (5,6) but the coefficient A is no longer constant⁴, it depends upon the radius of the collapsing core r . We can deduce the dependence from infinite conductivity of the star and continuity of the normal component of the magnetic field on the surface of the core. In our case it can be written in the form (see Ref. 4, equation(24) for more details): $B_r r^2 \sin^2 \theta = \text{const}$. Since the collapse is radial, θ coordinate of a surface element considered remains constant during it and, consequently, $(B_r r^2)$ and $(B_r r^2 / \cos \theta)$ are also constant. We define a new quantity $D \equiv B_r \frac{r^2}{r_g^2 \cos \theta}$ which thus remains constant during the collapse (in contrast to A).

It should be derived from the initial conditions. We obtain from (5):

$$A \left[-\frac{r^2}{r_g^2} \ln \left(1 - \frac{r_g}{r} \right) - \frac{r}{r_g} - \frac{1}{2} \right] = D \quad (7)$$

$$A = D \left[-\frac{r^2}{r_g^2} \ln \left(1 - \frac{r_g}{r} \right) - \frac{r}{r_g} - \frac{1}{2} \right]^{-1} \quad (8)$$

Substituting this A into (5) and (6), we get the formulas describing the magnetic field around the collapsing core:

$$B_r = D \frac{r_g^2}{r^2} \cos \theta \quad (9)$$

$$B_\theta = D \sin \theta \sqrt{\frac{r_g^2}{r(r - r_g)}} \left(\frac{\left(\frac{r}{r_g} - 1\right) \ln\left(1 - \frac{r_g}{r}\right) + 1 - \frac{r_g}{2r}}{-\left(\frac{r}{r_g}\right)^2 \ln\left(1 - \frac{r_g}{r}\right) - \frac{r}{r_g} - \frac{1}{2}} \right) \quad (10)$$

3.2. Calculation of the charge created by the collapsing core rotation in the magnetic field

Tangential velocity of a particle falling on a black hole is defined by the angular momentum conservation condition⁵. For a non-relativistic particle falling in the equatorial plane with small angular momentum \mathfrak{M} we have:

$$v_\phi = \frac{\alpha c}{r} \sqrt{\frac{r - r_g}{r}}$$

Here $\alpha \equiv \frac{\mathfrak{M}}{mc}$ is a constant. Since the angular momentum of the collapsing core also remains constant, by analogy we can suggest the tangential velocity of the surface to be

$$v_\phi = \frac{\alpha c}{r} \sqrt{\frac{r - r_g}{r}} \sin \theta \quad (11)$$

Here $\sin \theta$ provides solid body rotation of the surface, and α is as before a constant which should be derived from the initial conditions.

In order to calculate electric field intensity, we should make some assumptions about the substance surrounding the collapsing core. It is most likely hot but respectively rare plasma. As it was already mentioned, we consider it as a perfect conductor. Since the magnetic field of the core is very strong at the late stages of the collapse, we can expect that the plasma is frozen to the magnetic field and corotates. Consequently, its tangential velocity v_ϕ (just around the surface of the core) is equal to (11). The question of v_r and v_θ components of the plasma velocity is not so clear. We speculate that they are equal to zero. Properly speaking, in the nonrelativistic case they cannot make a contribution to the electric field flux (since $B_\phi = 0$), but if they are relativistic, they contributes to the v^2 in (1). Hence, in our case the supposition $v_r = 0$, $v_\theta = 0$ means only that the velocities are much less than the speed of light.

Finally, for the substance velocity we have:

$$v_r = 0 \quad v_\theta = 0 \quad v_\phi = \frac{\alpha c}{r} \sqrt{\frac{r - r_g}{r}} \sin \theta \quad (12)$$

Since we presume the rotation of the core to be slow, the ratio α/r_g is small and $v_\phi \ll c$; in any case near the gravitational radius $v_\phi \rightarrow 0$. We can therefore use (2)

to calculate the electric field. Combining (10) and (12), we obtain for the electric field just above the core surface:

$$E_r = \frac{\alpha D r_g}{r^2} \left(\frac{\left(\frac{r}{r_g} - 1\right) \ln\left(1 - \frac{r_g}{r}\right) + 1 - \frac{r_g}{2r}}{-\left(\frac{r}{r_g}\right)^2 \ln\left(1 - \frac{r_g}{r}\right) - \frac{r}{r_g} - \frac{1}{2}} \right) \sin^2 \theta$$

Integrating it over the sphere surrounding the core and using equation (3), we obtain for the charge:

$$Q = \frac{\alpha D r_g}{3\pi} \left(\frac{\left(\frac{r}{r_g} - 1\right) \ln\left(1 - \frac{r_g}{r}\right) + 1 - \frac{r_g}{2r}}{-\left(\frac{r}{r_g}\right)^2 \ln\left(1 - \frac{r_g}{r}\right) - \frac{r}{r_g} - \frac{1}{2}} \right) \quad (13)$$

The charge is positive if the angular velocity of the core and its magnetic moment are codirectional.

4. Discharge processes and the new-born black hole charge estimation

The charge described by (13), reaches its maximum at $r \simeq 1.3r_g$ and drops to zero as $|\ln((r - r_g)/r_g)|^{-1}$ when r tends to r_g . Seemingly, it should mean that the charge of the formed black hole is zero. However, we have not yet taken into account gravitational force and its influence on the discharge processes. Let us consider a resting probe particle of mass m and charge q (let us denote its specific charge by μ). Electric field acting on the particle consists of several components: electric field (1) produced by the plasma rotation in the magnetic field and electrostatic field created by the excess of nonequilibrium charge (with respect to (13)) which has not discharged. Magnetic field impedes the discharge (equilibrium quantity of charge, described by (13), is always nonzero). However, since the equilibrium charge approaches zero when $r \rightarrow r_g$, we can ignore for simplicity the influence of the magnetic field at the last stages of the collapse.

The electrostatic force acting on the probe particle can be taken as:

$$F_e = \frac{qQ}{r^2} \quad (14)$$

Gravitational force acting on the particle is^{8,9}:

$$F_g = -\frac{GMm}{r^2 \sqrt{1 - \frac{r_g}{r}}} \quad (15)$$

Both the forces directed radially. The discharge (by particles with specific charge

μ) will be stopped by the gravitational field, when $F_e + F_g = 0$.

$$\begin{aligned} \frac{qQ}{r^2} &= \frac{GMm}{r^2\sqrt{1-\frac{r_g}{r}}} \\ Q &= \frac{r_g c^2}{2\mu} \sqrt{\frac{r}{r-r_g}} \end{aligned} \quad (16)$$

The last equation represents the charge which can be held by the gravitational field. It is represented on *Fig. 1* by the dot line)^b. The equilibrium charge described by (13) is represented by the solid line. The graphs have the only crossing point. When it is reached, the discharge is stopped by the gravitational field. Consequently, we can take the charge of the collapsar at this point as an estimation of the charge of new-born black hole. We equate expressions (13) and (16):

$$\frac{\alpha Dr_g}{3\pi} \left(\frac{\left(\frac{r}{r_g}-1\right) \ln\left(1-\frac{r_g}{r}\right) + 1 - \frac{r_g}{2r}}{-\left(\frac{r}{r_g}\right)^2 \ln\left(1-\frac{r_g}{r}\right) - \frac{r}{r_g} - \frac{1}{2}} \right) = \frac{r_g c^2}{2\mu} \sqrt{\frac{r}{r-r_g}}$$

Introducing a new dimensionless constant

$$\mathfrak{Y} = \frac{3\pi c^2}{2\alpha\mu D} \quad (17)$$

we finally obtain:

$$\left(\frac{\left(\frac{r}{r_g}-1\right) \ln\left(1-\frac{r_g}{r}\right) + 1 - \frac{r_g}{2r}}{-\left(\frac{r}{r_g}\right)^2 \ln\left(1-\frac{r_g}{r}\right) - \frac{r}{r_g} - \frac{1}{2}} \right) = \mathfrak{Y} \sqrt{\frac{r}{r-r_g}} \quad (18)$$

In order to find the charge we should solve this equation and substitute the obtained radius r into (16) or (13). If \mathfrak{Y} is very small ($-\ln \mathfrak{Y} \gg 1$), then $-\ln(r/r_g - 1) \gg 1$, and the equation can be simplified:

$$\frac{1}{2\mathfrak{Y}} = \sqrt{\frac{r_g}{r-r_g}} \ln \frac{r_g}{r-r_g} \quad (19)$$

5. Application of the results for calculation of the charge of an astronomical black hole

Let us estimate the electric charge of a real new-born black hole. We have already mentioned that a sun mass black hole is born by the collapse of a massive star

^bActually, the figure represents the left (solid line) and the right (dot line) parts of the equation (18), but they are proportional to the charge created by the collapsing core rotation in the magnetic field and to the charge which can be held by the gravitational field, respectively, with the coefficient depending upon the parameters of the system.

central part. As it is believed, neutron stars are also born as a consequence of massive star core collapses, but of lower masses. Consequently, the characteristic physical parameters of the core can be estimated from the parameters of neutron stars. The radius of a neutron star is $10 - 15$ km, the mass is $1.4 - 1.8M_{\odot}$. Pulsars have the magnetic field $10^{12} - 10^{13}$ Gs, the shortest known period of revolution⁶ of a pulsar is 1.5 ms.

We take the mass of the collapsing core $3M_{\odot}$ ($r_g = 9 \cdot 10^5$ cm). It is heavier than a neutron star to form a black hole. Hence, we should consider its physical parameter when its radius is bigger. Using the above-mentioned analogy, we presume that when the radius of the collapsing core is 20 km its period of revolution is 10^{-3} s and the magnetic field intensity on its pole is 10^{13} Gs. As the specific charge μ we take the specific charge of electron (or positron). If the charge Q is negative (the angular velocity of the core and its magnetic moment are oppositely directed), it is obvious. But even in the case when the charge of the core is positive, the temperature at the last stages of the collapse is so high that the positron concentration is sufficient to provide the discharge. From this we calculate α, D and \mathfrak{Y} . The formula for the period of revolution coincides with the classical one: $T_{rev} = \frac{2\pi r}{cv_{\phi}}$. Subtracting v_{ϕ} from (12), we finally find $\alpha \simeq 11$ km.

We find $\mathfrak{Y} = 1.4 \cdot 10^{-16}$. As soon as $-\ln \mathfrak{Y} \simeq 36 \gg 1$ we can use (19). Solving the equation, we have:

$$\sqrt{\frac{r - r_g}{r_g}} = 1.77 \cdot 10^{-14} \quad (20)$$

$$Q = 1.4 \cdot 10^{13} \text{ Coulombs} \quad (21)$$

The maximum charge of the core is reached (in accordance with (13)), when the radius is $r \simeq 1.3r_g$. It is equal to $Q_m = 4.3 \cdot 10^{14}$ Coulombs, thirty times higher than the final one.

6. Discussion

The foregoing calculations may provoke several objections. First of all, the radius (20), where the discharge is stopped by the gravitational field, is too close to the Schwarzschild sphere. The disturbance of the sphere produced by the core rotation is much stronger, and the adduced solution is certainly not valid for this region. Moreover, we used a lot of strong simplifying suppositions, such as the calculation of the magnetic field on the assumption of quasistatic collapse. These remarks are correct, but they do not affect significantly the conclusions of the article. The purpose of the work was to show that a new-born black hole can possess a big electric charge. The charge of the core surpasses 10^{14} C since the radius of the collapsing core becomes less than $r \simeq 8r_g$, where the departure of the metric from the Schwarzschild one is not so significant. Moreover, the final charge depends weakly on the parameter (20) and, consequently, on the details of the discharge processes.

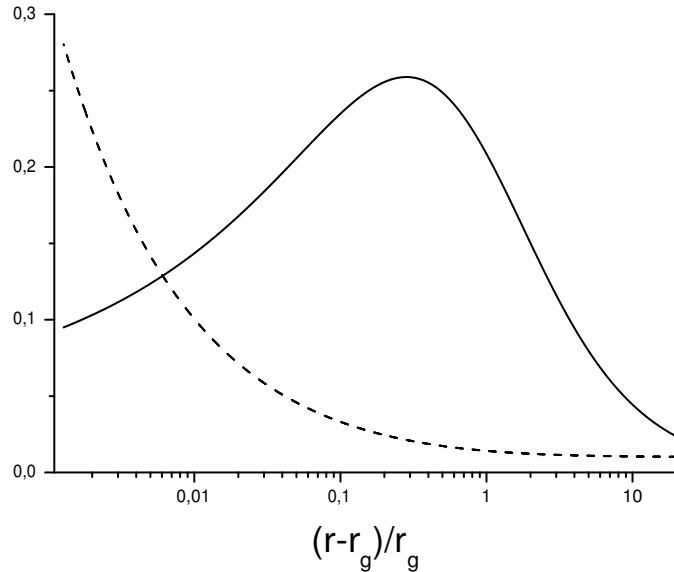


Fig. 1. represents the left (solid line) and the right (dot line) parts of equation (18) for the case when $\mathfrak{Y} = 0.01$. They are proportional to the charge created by the collapsing core rotation in the magnetic field and to the charge which can be held by the gravitational field, respectively, with the coefficient depending upon the parameters of the system.

Of course, the estimation (21) can be inaccurate, but nevertheless we have strong reasons to presume the final charge to be bigger than 10^{13} C.

Initially, the star is not electrically charged. As we have shown, because of the magnetodynamical processes a charge separation appears. Then what happens with the charge separated from the forming black hole? Of course, it is emitted to the substance of the envelope; a further destiny of the charge depends on the envelope evolution. The collapse of a core of a massive star is considered numerically in Ref. 7. It is shown that the differential rotation of the envelope appears resulting in the magnetic field generation. At some moment magnetohydrodynamical instability arises; it leads to the twisting of the magnetic lines and closed magnetic vortex generation. The magnetic field structure becomes very complex. Finally, magnetic pressure increases so that it ejects the envelope to the outer space producing a supernova explosion. The charge evolved from the core moves along the magnetic lines and is trapped by the forming vortices. During the supernova explosion the envelope substance is erupted out together with the charge frozen in the magnetic vortices.

In the *model description* we presumed that the core and the substance surrounding the core have infinitive conductivity. This statement can be proved numerically.

Conductivity of the totally ionized plasma depends only on its temperature and does not depend on the density⁶:

$$\sigma = 8 \cdot 10^{-4} T_e^{3/2} \left[\frac{1}{\text{Ohm} \cdot \text{m}} \right]$$

Let us consider a sphere of the radius of 20 km (characteristic for the collapsing core) and with electric charge $1.4 \cdot 10^{13}$ C merged into the plasma with the temperature $T_e = 10^6$ K. Then electric field intensity around the sphere is $6 \cdot 10^{14}$ V/m, current density is $5 \cdot 10^{20}$ A/m², and the total current is $2.5 \cdot 10^{30}$ A. The sphere would be discharged in $\sim 10^{-17}$ s. Of course, this time has not a direct physical meaning, but this proves the negligibility of the electrical resistance in the considered task.

It is important to notice that though the obtained charge of an astrophysical black hole is big, the charge to mass ratio is small $Q/(\sqrt{GM}) \sim 10^{-7}$, and it is not sufficient to affect significantly either the gravitational field of the star or the dynamics of its collapse. The numerical collapse of a charged star in the Reissner-Nordström space-time was computed^{10,11}. They considered spherical symmetry with distribution of an electric charge proportional to the distribution of mass. For the numerical computation they assumed a polytropic equation of state with $\gamma = 5/3$. They concluded that for low values of $Q/(\sqrt{GM})$ no departure from unpolarized collapse was found. In our case $Q/(\sqrt{GM}) \sim 10^{-7}$, which satisfies the criteria¹⁰, and it also justifies our approach of using a Schwarzschild metric.

In the article, we considered a collapsing core with a moderate magnetic field. The electric charge of the new-born black hole turns to be sizable, but its electrostatic field ($\sim 10^{12}$ Gs) is lower than the critical one ($4.4 \cdot 10^{13}$ Gs). However, in the case of magnetars, the magnetic field of the collapsing core can also be extremely strong ($\sim 10^{15}$ Gs). Then the charge of the black hole can be so big that the electric field will surpass the critical. In this case a new effect appears^{12,13}: a very large number of e^+e^- pairs is created in the region with overcritical electric field, which finally leads to the formation of a strong gamma-ray burst.

Interaction of the accreting substance with the energetic γ -ray emission of a black hole can also induce an electric charge into it¹⁴. The Compton scattering of γ -photons kicks out electrons from the accretion flow, while for the protons this mechanism is not so effective because of higher mass-charge ratio. As a result, the black hole should obtain a significant charge. However, one can see¹⁴ that in the system considered the charge induced by this process is approximately 100 times lower than (21). Actually, in our case the mechanism is not effective.

In Ref. 15, processes in the force-free magnetosphere of an already formed black hole are considered. As it was shown, because of accretion the black hole acquires a significant electric charge:

$$Q \sim 10^{15} \text{C} \left(\frac{B}{10^{15} \text{Gs}} \right) \left(\frac{M}{M_\odot} \right)^2$$

Here B is the magnetic field intensity in the accreting substance near the black hole. This quantity is big and quite correlates with (21). Thus, an astrophysical

black hole can possess a big electric charge, either getting it during the birth or owing to the accretion.

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